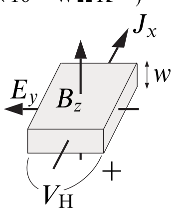
6.5 Electrons in solids

Free electron transport properties

Current density	$\boldsymbol{J} = -ne\boldsymbol{v}_{\mathrm{d}}$	(6.60)	$\boldsymbol{J}$ current density n free electron number density $-e$ electronic charge $\boldsymbol{v}_{\mathrm{d}}$ mean electron drift velocity τ mean time between collisions (relaxation time) m_{e} electronic mass
Mean electron drift velocity	$\boldsymbol{v}_{\mathrm{d}} = -\dfrac{e\tau}{m_{\mathrm{e}}}\boldsymbol{E}$	(6.61)	
d.c. electrical conductivity	$\sigma_0 = \dfrac{ne^2\tau}{m_{\mathrm{e}}}$	(6.62)	$\boldsymbol{E}$ applied electric field σ_0 d.c. conductivity ($\boldsymbol{J} = \sigma\boldsymbol{E}$)
a.c. electrical conductivity[a]	$\sigma(\omega) = \dfrac{\sigma_0}{1 - \mathrm{i}\omega\tau}$	(6.63)	ω a.c. angular frequency $\sigma(\omega)$ a.c. conductivity
Thermal conductivity	$\lambda = \dfrac{1}{3}\dfrac{C_V}{V}\langle c^2\rangle\tau$	(6.64)	C_V total electron heat capacity, V constant V volume $\langle c^2\rangle$ mean square electron speed k_{B} Boltzmann constant T temperature T_{F} Fermi temperature
	$= \dfrac{\pi^2 n k_{\mathrm{B}}^2 \tau T}{3m_{\mathrm{e}}} \quad (T \ll T_{\mathrm{F}})$	(6.65)	
Wiedemann–Franz law[b]	$\dfrac{\lambda}{\sigma T} = L = \dfrac{\pi^2 k_{\mathrm{B}}^2}{3e^2}$	(6.66)	L Lorenz constant ($\simeq 2.45 \times 10^{-8}\,\mathrm{W}\,\Omega\,\mathrm{K}^{-2}$) λ thermal conductivity
Hall coefficient[c]	$R_{\mathrm{H}} = -\dfrac{1}{ne} = \dfrac{E_y}{J_x B_z}$	(6.67)	R_{H} Hall coefficient E_y Hall electric field J_x applied current density B_z magnetic flux density V_{H} Hall voltage I_x applied current ($= J_x \times$ cross-sectional area) w strip thickness in z
Hall voltage (rectangular strip)	$V_{\mathrm{H}} = R_{\mathrm{H}}\dfrac{B_z I_x}{w}$	(6.68)	



[a]For an electric field varying as $\mathrm{e}^{-\mathrm{i}\omega t}$.

[b]Holds for an arbitrary band structure.

[c]The charge on an electron is $-e$, where e is the elementary charge (approximately $+1.6 \times 10^{-19}$ C). The Hall coefficient is therefore a negative number when the dominant charge carriers are electrons.

Fermi gas

Electron density of states[a]	$g(E)=\frac{V}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}E^{1/2}$ $g(E_F)=\frac{3}{2}\frac{nV}{E_F}$	(6.69) (6.70)	E electron energy (>0) $g(E)$ density of states V "gas" volume m_e electronic mass $\hbar$ (Planck constant)/(2π)
Fermi wavenumber	$k_F=(3\pi^2 n)^{1/3}$	(6.71)	k_F Fermi wavenumber n number of electrons per unit volume
Fermi velocity	$v_F=\hbar k_F/m_e$	(6.72)	v_F Fermi velocity
Fermi energy ($T=0$)	$E_F=\frac{\hbar^2 k_F^2}{2m_e}=\frac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$	(6.73)	E_F Fermi energy
Fermi temperature	$T_F=\frac{E_F}{k_B}$	(6.74)	T_F Fermi temperature k_B Boltzmann constant
Electron heat capacity[b] ($T \ll T_F$)	$C_{Ve}=\frac{\pi^2}{3}g(E_F)k_B^2 T$ $=\frac{\pi^2 k_B^2}{2E_F}T$	(6.75) (6.76)	C_{Ve} heat capacity per electron T temperature
Total kinetic energy ($T=0$)	$U_0=\frac{3}{5}nVE_F$	(6.77)	U_0 total kinetic energy
Pauli paramagnetism	$\boldsymbol{M}=\chi_{HP}\boldsymbol{H}$ $=\frac{3n}{2E_F}\mu_0\mu_B^2\boldsymbol{H}$	(6.78) (6.79)	χ_{HP} Pauli magnetic susceptibility $\boldsymbol{H}$ magnetic field strength $\boldsymbol{M}$ magnetisation μ_0 permeability of free space μ_B Bohr magneton
Landau diamagnetism	$\chi_{HL}=-\frac{1}{3}\chi_{HP}$	(6.80)	χ_{HL} Landau magnetic susceptibility

[a]The density of states is often quoted per unit volume in real space (i.e., $g(E)/V$ here).
[b]Equation (6.75) holds for any density of states.

Thermoelectricity

Thermopower[a]	$\mathscr{E}=\frac{\boldsymbol{J}}{\sigma}+S_T\nabla T$	(6.81)	$\mathscr{E}$ electrochemical field[b] $\boldsymbol{J}$ current density σ electrical conductivity
Peltier effect	$\boldsymbol{H}=\Pi\boldsymbol{J}-\lambda\nabla T$	(6.82)	S_T thermopower T temperature $\boldsymbol{H}$ heat flux per unit area
Kelvin relation	$\Pi=TS_T$	(6.83)	Π Peltier coefficient λ thermal conductivity

[a]Or "absolute thermoelectric power."
[b]The electrochemical field is the gradient of $(\mu/e)-\phi$, where μ is the chemical potential, $-e$ the electronic charge, and ϕ the electrical potential.

Band theory and semiconductors

Bloch's theorem	$\Psi(\boldsymbol{r}+\boldsymbol{R})=\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{R})\Psi(\boldsymbol{r})$	(6.84)	Ψ electron eigenstate $\boldsymbol{k}$ Bloch wavevector $\boldsymbol{R}$ lattice vector $\boldsymbol{r}$ position vector
Electron velocity	$\boldsymbol{v}_b(\boldsymbol{k})=\frac{1}{\hbar}\nabla_{\boldsymbol{k}}E_b(\boldsymbol{k})$	(6.85)	$\boldsymbol{v}_b$ electron velocity (for wavevector $\boldsymbol{k}$) $\hbar$ (Planck constant)/2π b band index $E_b(\boldsymbol{k})$ energy band
Effective mass tensor	$m_{ij}=\hbar^2\left[\frac{\partial^2 E_b(\boldsymbol{k})}{\partial k_i\partial k_j}\right]^{-1}$	(6.86)	m_{ij} effective mass tensor k_i components of $\boldsymbol{k}$
Scalar effective mass[a]	$m^*=\hbar^2\left[\frac{\partial^2 E_b(k)}{\partial k^2}\right]^{-1}$	(6.87)	m^* scalar effective mass k $=\lvert\boldsymbol{k}\rvert$
Mobility	$\mu=\frac{\lvert\boldsymbol{v}_\mathrm{d}\rvert}{\lvert\boldsymbol{E}\rvert}=\frac{eD}{k_\mathrm{B}T}$	(6.88)	μ particle mobility v_d mean drift velocity $\boldsymbol{E}$ applied electric field $-e$ electronic charge D diffusion coefficient T temperature
Net current density	$\boldsymbol{J}=(n_\mathrm{e}\mu_\mathrm{e}+n_\mathrm{h}\mu_\mathrm{h})e\boldsymbol{E}$	(6.89)	$\boldsymbol{J}$ current density $n_\mathrm{e,h}$ electron, hole, number densities $\mu_\mathrm{e,h}$ electron, hole, mobilities
Semiconductor equation	$n_\mathrm{e}n_\mathrm{h}=\frac{(k_\mathrm{B}T)^3}{2(\pi\hbar^2)^3}(m_\mathrm{e}^*m_\mathrm{h}^*)^{3/2}\mathrm{e}^{-E_\mathrm{g}/(k_\mathrm{B}T)}$	(6.90)	k_B Boltzmann constant E_g band gap $m_\mathrm{e,h}^*$ electron, hole, effective masses
p-n junction	$I=I_0\left[\exp\left(\frac{eV}{k_\mathrm{B}T}\right)-1\right]$ $I_0=en_\mathrm{i}^2A\left(\frac{D_\mathrm{e}}{L_\mathrm{e}N_\mathrm{a}}+\frac{D_\mathrm{h}}{L_\mathrm{h}N_\mathrm{d}}\right)$ $L_\mathrm{e}=(D_\mathrm{e}\tau_\mathrm{e})^{1/2}$ $L_\mathrm{h}=(D_\mathrm{h}\tau_\mathrm{h})^{1/2}$	(6.91) (6.92) (6.93) (6.94)	I current I_0 saturation current V bias voltage (+ for forward) n_i intrinsic carrier concentration A area of junction $D_\mathrm{e,h}$ electron, hole, diffusion coefficients $L_\mathrm{e,h}$ electron, hole, diffusion lengths $\tau_\mathrm{e,h}$ electron, hole, recombination times $N_\mathrm{a,d}$ acceptor, donor, concentrations

[a]Valid for regions of k-space in which $E_b(\boldsymbol{k})$ can be taken as independent of the direction of k.